

Singular and Regular Gauges in Soft Collinear Effective Theory: The Introduction of the New Wilson Line T

Ahmad Idilbi^{1,*} and Ignazio Scimemi^{1,†}

¹*Departamento de Física Teórica II,
Universidad Complutense de Madrid (UCM), 28040 Madrid, Spain*

Abstract

Gauge invariance in soft-collinear effective theory (SCET) is discussed in regular (covariant) and singular (light-cone) gauges. It is argued that SCET, as it stands, is not capable to define in a gauge invariant way certain non-perturbative matrix elements that are an integral part of many factorization theorems. Those matrix elements involve two quark or gluon fields separated not only in light-cone direction but also in the transverse one. This observation limits the range of applicability of SCET. To remedy this we argue that one needs to introduce a new Wilson line as part of SCET formalism, that we call T . This Wilson line depends only on the transverse component of the gluon field. As such it is a new feature to the SCET formalism and it guarantees gauge invariance of the non-perturbative matrix elements in both classes of gauges.

*Electronic address: ahmadidilbi@gmail.com

†Electronic address: ignazios@fis.ucm.es

In the era of the Large Hadron Collider (LHC) where two hadron beams collide, factorization theorems of the various cross sections of phenomenological interest are the basis of any phenomenological analysis. In recent years, Soft-Collinear Effective Theory (SCET) [1–3] has emerged as the most effective framework within which such factorization theorems can be established in a rather coherent manner. The success of SCET to factorize high-energy processes (and thereby allowing resummation of large logarithms) rests mainly on two factors. The first is the de-coupling of the soft and collinear modes, responsible of the infra-red (IR) divergences in perturbative QCD, at the level of the SCET Lagrangian and operators. Identifying those IR divergences, essentially, is the most important step in establishing factorization theorems since it allows for an operator definition of the non-perturbative hadronic matrix elements. The second factor is the *gauge invariant* building-blocks of SCET which render the previous operator definitions gauge invariant as they should be. In SCET there are two fundamental quantities with which hadronic matrix elements can be defined. In the quark-gluon collinear sector we have $W_n^\dagger \xi_{\bar{n}}$ and $W_{\bar{n}} G^{\mu\nu} W_{\bar{n}}$ for gluon-gluon collinear interactions. It is well-known that those quantities are gauge invariant (under collinear and soft gauge transformations) as long as the gauge transformation is unity at infinity (in other words $\alpha^a(x^- = \infty) = 0$ where $\alpha^a(x)$ is the gauge transformation matrix.) This is the case in covariant gauges.

The covariant gauge is “regular” in the sense that all components of the gluon field A^μ vanish at light-cone infinity while for “singular” gauge, certain components do not. Actually one can show that in QED in light-cone gauge we have the following [4],

$$A^+ = A^- = 0, \quad A_\perp = -\frac{e}{2\pi} \theta(x^-) \nabla \ln \mu r_\perp, \quad (1)$$

where e is the electric charge, x^- is a light-cone coordinate and r_\perp is the distance in the transverse direction. μ is an arbitrary mass scale needed for dimensional purposes.

The fact that in light-cone gauge A_\perp does not vanish at $x^- = \infty$ (unlike the case in covariant gauge) will have a rather very important impact on the “gauge invariant” SCET matrix elements as we shall see below. The basic observation is that in singular gauges a gauge transformation at infinity in light-cone direction, can be applied on the quark field which has to be, as usual, compensated for by a Wilson line. The result of this simple observation is that in light cone gauge one is enforced to introduce a new Wilson lines, built only from the transverse components A_\perp^μ . We will call these Wilson lines T , T^\dagger as they contain only the transverse components of the gluon field. The T -Wilson line involves only the transverse component of the gluon field. It is a new feature to be added to the definition of SCET matrix elements.

There are two main results for the introduction of the T -Wilson line. The first is that SCET will not be altered once we work in covariant gauges since $T = 1$ in that case. However, and more importantly, it will allow for gauge invariant definitions of the non-perturbative matrix elements when the two quark or gluon fields are separated not only in the x^- direction but also in the transverse ones as well. As an example, consider the transverse-momentum-dependent parton distribution function (TMDPDF). This function enters the factorization theorem of the semi-inclusive deep-inelastic scattering (SIDIS) processes, see Ref. [5, 6]. The SCET version of the TMDPDF, built only from the collinear quark field and the collinear

Wilson line, is not gauge invariant. As such the SCET TMDPDF is not really a genuine physical quantity. However when the contribution from the T -Wilson line is taken into account it becomes gauge invariant in both classes of gauges: regular and singular ones. More discussion on this will be given below and in a forthcoming paper [7].

An important piece of discussion in light-cone gauge concerns the type of prescription to be used in the gluon propagator. In Ref. [9] it was shown that the canonical quantization of QCD in light-cone gauge is consistent only with the Mandelstam-Leibbrandt (ML) prescription. This prescription is the only one which respects causality, unitarity, QCD power counting and allows for Wick rotations in loop integrals. Clearly, this prescription must be taken as the reference prescription also in SCET. The use of a particular prescription is related to the regularization of infrared (IR) divergences, as the prescription regulates essentially infrared singularities. Thus the use of prescriptions different from ML's can be justified in some cases with a compatible regularization of IR divergences. We discuss this issue in more detail below.

I. THE SCET JET IN FEYNMAN GAUGE AND IN LIGHT-CONE GAUGE WITH ML PRESCRIPTION

Let us consider the fundamental quantity in SCET $\langle 0 | W_{\bar{n}}^\dagger \xi_{\bar{n}} | q_{\bar{n}} \rangle$ which describes an incoming parton moving along the $\bar{n} = \frac{1}{\sqrt{2}}(1, 0, 0)$ direction (our notation is $(+, -, \perp)$ for light-cone coordinates). We will demonstrate below, at the one-loop level, that this quantity, with the zero-bin subtracted [8] (i.e. the pure collinear jet) is not gauge invariant. It will attain different results in Feynman gauge and in light-cone gauge. It should be mentioned however that the last statement is true depending on the prescription used to regularize the spurious “light-cone singularity” originating from the $1/k^+$ term in light-cone gauge when it approaches 0. However when the contribution from the T -Wilson line is taken into account, this prescription-dependence cancels and gauge invariance is completely restored. In

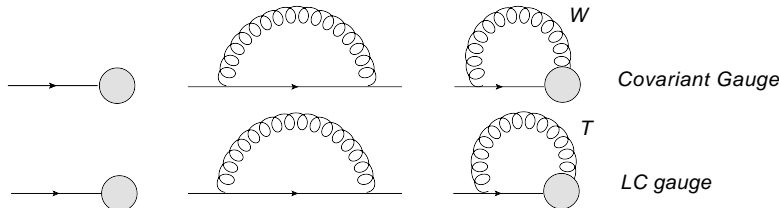


FIG. 1: The matrix element for an incoming jet at tree level and one loop in SCET in covariant and LC gauge

Feynman gauge the two diagrams contributing at one-loop level are given in the first line of Figure I. We work in dimensional regularization and we take the external quark momentum $p = (p^+, 0, \mathbf{p}_\perp)$ to be off-shell, $p^2 = -\mathbf{p}_\perp^2 \neq 0$, $p^+ > 0$. Let us denote the contribution from

the wave function renormalization graph (WFR) by Σ_{Fey} (see e.g. the second reference in [1].) The contribution from the diagram with collinear Wilson line is given by

$$I_{\bar{n},Fey} = -2ig^2 C_F \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i0)(k^+ + i0)} \frac{p^+ + k^+}{(p+k)^2 + i0} , \quad (2)$$

where $C_F = 4/3$. The gluon momentum has been chosen such that it is outgoing from the vertex with the collinear Wilson line. The calculation of this integral and others as well are given in the Appendix (see also [6, 10].) It is important to note that in principle this integral contains a real and an imaginary part. The imaginary part of the integral originates from the divergence when $k^+ \rightarrow 0$. This imaginary part does not appear if one insists on using dimensional regularization to regulate IR divergences (see the Appendix).

In light-cone gauge, the choice of the gauge condition is $n \cdot A_{\bar{n}} \equiv A_{\bar{n}}^+ = 0$ where $n = \frac{1}{\sqrt{2}}(0, 1, 0)$. Clearly this choice renders $W_{\bar{n}} = 1$. In light-cone gauge and without the contribution from the T -Wilson line there is only the WFR graph. The gluon propagator in this gauge is given by

$$\tilde{D}_{\mu\nu}(k) = \frac{-i}{k^2 + i0} \left(g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{[k^+]} \right) , \quad (3)$$

where the symbol $[k^+]$ means that an appropriate prescription condition must be chosen to regularize the spurious light-cone singularity. The contribution from $g_{\mu\nu}$ in Eq. (3) to the WFR gives the same result as in Feynman gauge. The remaining contribution is known as the “Axial part”. Thus the final result in light-cone gauge is given by

$$\Sigma_{LC}(p) = \Sigma_{Fey}(p) + \Sigma_{Ax}^{(Pres)}(p) = (I_{w,Fey}(p) + I_{w,Ax}^{(Pres)}(p)) \frac{ip^2}{p^+} \frac{\not{n}}{\sqrt{2}} , \quad (4)$$

where the suffix “Pres” indicates the prescription dependence of the integral. The most common prescriptions used in full QCD calculations are listed in Tab. I. The value of $I_{w,Fey}(p)$

Prescription	$1/[k^+]$
$+i0$	$1/(k^+ + i0)$
$-i0$	$1/(k^+ - i0)$
PV	$1/2(1/(k^+ + i0) + 1/(k^+ - i0))$
ML	$1/(k^+ + i0 \text{Sgn}(k^-))$

TABLE I: The most commonly used prescriptions light-cone gauge.

can be easily extracted from that of Σ_{Fey} and we do not need it explicitly here. It is also well-known that $I_{w,Fey}(p)$ is the same in SCET and in QCD. This is no more true for the axial part, $I_{w,Ax}^{(Pres)}(p)$, whose result of calculation is different in the two cases. In SCET one

finds

$$\begin{aligned}\Sigma_{Ax}^{(\text{Pres})}(p) &= I_{w,Ax}^{(\text{Pres})}(p) \frac{\not{n}}{\sqrt{2}} \frac{ip^2}{p^+} \\ &= 4g^2 C_F \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{-i}{(k^2 + i0)[k^+]} \left(\frac{p^+}{p^2} - \frac{p^+ + k^+}{(p+k)^2 + i0} \right) \frac{ip^2}{p^+} \frac{\not{n}}{\sqrt{2}}.\end{aligned}\quad (5)$$

The first integral is identically zero in all prescriptions that we have studied so we end up with

$$I_{w,Ax}^{(\text{Pres})}(p) = 4ig^2 C_F \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i0)[k^+]} \frac{p^+ + k^+}{(p+k)^2 + i0}.\quad (6)$$

The gauge invariance of the SCET matrix element is guaranteed when $I_{\bar{n},Fey} = (-1/2)I_{Ax}^{(+i0)}$. In ML prescription we have

$$\frac{1}{[k^+]} = \frac{\theta(k^-)}{k^+ + ip^+\eta} + \frac{\theta(-k^-)}{k^+ - ip^+\eta}.\quad (7)$$

where $\eta > 0$ is a small number and is used to regulate the $k^+ \rightarrow 0$ divergence. In the ML prescription one finds that in Eq. (6) only the integrand with k^+ in the numerator gives a non-zero result with the off-shellness that we have chosen (see e.g. [10]) and the result is real and independent of η however, and most importantly, it has only a single pole. The integral with p^+ in the numerator gives zero in ML prescription. Subtracting the zero-bin contribution has also no effect as it is zero in the ML prescription (see Appendix) for the off-shellness that we have chosen. Since the pure jet function in Feynman gauge has a double pole one can easily conclude that the statement that the matrix elements: $\langle 0|W_{\bar{n}}^\dagger \xi_{\bar{n}}|q\rangle$ and $\langle q|\bar{\xi}_n W_n|0\rangle$ are gauge invariant in both type of gauges, regular and singular ones, is in general not true.

Let us now consider the matrix element describing an incoming jet

$$\langle 0|T_{\bar{n}}^\dagger W_{\bar{n}}^\dagger \xi_{\bar{n}}|q_{\bar{n}}\rangle,\quad (8)$$

where the T -Wilson line is

$$T_{\bar{n}}^\dagger(x^+, x_\perp) = \mathcal{P} \exp \left[ig \int_0^\infty d\tau \mathbf{l}_\perp \cdot \mathbf{A}_\perp(\infty^-, x^+; \mathbf{l}_\perp \tau + \mathbf{x}_\perp) \right].\quad (9)$$

The vector \mathbf{l} specifies a path in a two-dimensional space. Notice that in light-cone gauge and at infinity in one light-cone direction, the gauge field is a gradient of a scalar function thus the final result of integration is path-independent. In the following we study some of the properties of the new Wilson line $T_{\bar{n}}^\dagger$ and we show how it fixes gauge invariance issues discussed earlier.

First we note that $\mathbf{A}_\perp(\infty^-, x^+; \mathbf{l}_\perp) = 0$ in covariant gauges. So $T_{\bar{n}}^\dagger \equiv 1$ and $\langle 0|T_{\bar{n}}^\dagger W_{\bar{n}}^\dagger \xi_{\bar{n}}|q_{\bar{n}}\rangle$ reduces to $\langle 0|W_{\bar{n}}^\dagger \xi_{\bar{n}}|q_{\bar{n}}\rangle$ as it should be. This implies that all the results obtained in SCET in covariant gauges are still valid even with the introduction of the T -Wilson line. Then we use

$$\mathbf{A}_\perp(\infty^-, x^+; \mathbf{l}_\perp \tau) = \int \frac{dk^+}{2\pi} e^{-i[k^+]\infty^-} \int \frac{dk^-}{2\pi} e^{-i(k^-)x^+} \int \frac{dk_\perp^2}{(2\pi)^2} e^{i\mathbf{k}_\perp \cdot \mathbf{l}_\perp \tau} \mathbf{A}_\perp(k),\quad (10)$$

so that

$$\begin{aligned} \int_0^\infty d\tau \mathbf{l}_\perp \cdot \mathbf{A}_\perp(\infty^-, x^+; \mathbf{l}_\perp \tau) = \\ \int \frac{dk^+}{2\pi} e^{-i[k^+]\infty^-} \int \frac{dk^-}{2\pi} e^{-i(k^-)x^+} \int \frac{dk_\perp^2}{(2\pi)^2} \frac{i\mathbf{l}_\perp \cdot \mathbf{A}_\perp(k)}{\mathbf{k}_\perp \cdot \mathbf{l}_\perp - i0} . \end{aligned} \quad (11)$$

The fundamental observation now is that

$$\frac{e^{-i[k^+]\infty^-}}{[k^+]} = \frac{e^{i[k^+]\infty}}{[k^+]} = 2\pi i C_\infty^{(\text{Pres})} \delta(k^+) = \left[\frac{C_\infty^{(\text{Pres})}}{k^+ - i0} - \frac{C_\infty^{(\text{Pres})}}{k^+ + i0} \right] . \quad (12)$$

The $C_\infty^{(\text{Pres})}$ depends on the prescription that is chosen in light-cone gauge as is shown in Tab. (II).¹

Prescription	$C_\infty^{(\text{Pres})}$
$+i0$	0
$-i0$	1
PV	1/2
ML	$\theta(-k^-)$

TABLE II: $C_\infty^{(\text{Pres})}$ in the most used prescriptions for LC gauge

In the calculation of loops with T_n^\dagger we use then the following propagators. When μ picks up collinear indexes, $\mu = +, -$, and using Eq. (12) one finds

$$\begin{aligned} \langle A_\perp^i(\infty^-, x^+, x_\perp) A^\mu(0) \rangle &= \lim_{x^- \rightarrow -\infty} \int \frac{d^4 k}{(2\pi)^4} e^{-i([k^+]x^- + k^-x^+ + k_\perp x_\perp)} \frac{i}{k^2 + i0} \left(\frac{n^\mu k^i}{[k^+]} \right) \\ &= \int \frac{dk^-}{(2\pi)} C_\infty^{(\text{Pres})} e^{-ik^-x^+} \int \frac{d^2 k_\perp}{(2\pi)^2} e^{-ik_\perp x_\perp} \frac{1}{k_\perp^2 + i0} (-n^\mu k^i) \\ &= i \int \frac{d^4 k}{(2\pi)^4} \frac{e^{-i(k^-x^+ + k_\perp x_\perp)} (n^\mu k^i)}{k^2 + i0} \left[\frac{C_\infty^{(\text{Pres})}}{k^+ - i0} - \frac{C_\infty^{(\text{Pres})}}{k^+ + i0} \right] . \end{aligned} \quad (13)$$

where the last line of Eq. (13) is a useful form for practical calculations. When $\mu = j$ the

¹ The difference in $C_\infty^{(\text{Pres})}$ with respect to Ref. [6] depend on the fact that in SCET the Feynman rule for the vertex coming from the W^\dagger Wilson line has a $+i0$, while in Ref. [6] they consider the case of retarded Green functions.

same reasoning as in Eq. (13) gives

$$\begin{aligned}
\langle A_\perp^i(\infty^-, x^+, x_\perp) A^j(0) \rangle &= -\delta^{ij} \lim_{x^- \rightarrow -\infty} \int \frac{d^4 k}{(2\pi)^4} e^{-i([k^+]x^- + k^-x^+ + k_\perp x_\perp)} \frac{-i}{k^2 + i0} \\
&= -\delta^{ij} \lim_{x^- \rightarrow -\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{e^{-i([k^+]x^- + k^-x^+ + k_\perp x_\perp)}}{k^+} \frac{-ik^+}{k^2 + i0} \\
&= -\delta^{ij} \int \frac{d^4 k}{(2\pi)^4} e^{-i(k^-x^+ + k_\perp x_\perp)} \frac{k^+ \delta(k^+)}{k^2 + i0} C_\infty^{(\text{Pres})} = 0 . \quad (14)
\end{aligned}$$

Using these rules one can now calculate the one-loop contribution coming from T_n^\dagger ,

$$\begin{aligned}
I_{T, Ax}^{(\text{Pres})} &= 2C_F g^2 \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{-l_\perp^\mu}{l_\perp k_\perp} \frac{i(p^+ + k^+)}{(p + k)^2} \\
&\times i \left[\bar{n}^\nu + \frac{\gamma_\perp^\nu \not{p}_\perp}{p^+} + \frac{(\not{k}_\perp + \not{p}_\perp) \gamma_\perp^\nu}{p^+ + k^+} - \frac{(\not{k}_\perp + \not{p}_\perp) \not{p}_\perp}{p^+(p^+ + k^+) n^\nu} \right] \\
&\times \frac{i(n^\nu k^\mu)}{k^2 + i0} C_\infty^{(\text{Pres})} \left[\frac{1}{k^+ - i0} - \frac{1}{k^+ + i0} \right] \\
&= 2C_F g^2 \mu^{2\varepsilon} i \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{(k^2 + i0)((p + k)^2 + i0)} \left[\frac{C_\infty^{(\text{Pres})}}{k^+ - i0} - \frac{C_\infty^{(\text{Pres})}}{k^+ + i0} \right] . \quad (15)
\end{aligned}$$

The last line of Eq. (15) shows explicitly that the path-dependence of the T -Wilson line is canceled. This is to be expected since at infinity in light-cone direction and in light-cone gauge the gluon field is just a gradient of a scalar function so the line integral is path-independent. We expect that path independence is a general feature of the T -Wilson line, as the cancellation of prescription dependence is in principle independent of any path. A verification of this statement with the calculation of the matching of the electromagnetic form factor between SCET and QCD in light-cone gauge is in progress [7]. Summing up the last result with the axial contribution from the wave function renormalization diagram we find that gauge invariance is established when

$$I_{\bar{n}, Fey} = \frac{-1}{2} I_{w, Ax}^{(\text{ML})} + I_{T, Ax}^{(\text{ML})} . \quad (16)$$

Using the value of $C_\infty^{(\text{ML})}$ extracted from Eq. (12) and in Tab. II, one finds that all the prescription-dependence exactly cancels in the sum of the r.h.s. of Eq. (16) and the gauge invariance is realized. Eq. 16 is actually valid for all other prescriptions as well, $I_{\bar{n}, Fey} = -I_{w, Ax}^{(\text{Pres})}/2 + I_{T, Ax}^{(\text{Pres})}$, however zero-bin subtraction plays a special role in these other prescriptions as is discussed in the following section.

II. THE SCET JET IN $\pm i0$ AND PV PRESCRIPTIONS

The light-cone prescriptions $\pm i0$ and PV are in principle not compatible with the quantization of QCD in light-cone gauge [9]. Nevertheless one can ask what happens to the SCET jet when using these prescriptions in actual calculations. Comparing the integrands

in Eq. (2) (see the Appendix) and Eq. (6) we see that they agree when the prescription $[k^+] = k^+ + i\eta p^+$ is chosen. To show the difference between the $+i\eta$ and $-i\eta$ prescriptions it is enough at this stage to consider the ultra-violet (UV) divergent part of the integral $I_{w,Ax}^{(\text{Pres})}(p)$ where for simplicity of notation we label it as $I_{\bar{n}}^{(\text{Pres})}$ (the calculation of the finite part can be found in the Appendix). The result is,

$$I_{\bar{n}}^{\pm i\eta} = -2\frac{\alpha_s}{4\pi}C_F \left(\frac{\mu^2}{-p^2}\right)^\varepsilon \frac{1}{\varepsilon} \int_0^1 dz \frac{(1-z)^{1-\varepsilon} z^{-\varepsilon}}{z \mp i\eta}. \quad (17)$$

The singularity at $z = 0$ is regularized by the contour deformation. In this sense the $\pm i\eta$ serves to regularize the IR collinear divergence and thus we can Taylor expand the numerator in powers of ε . Keeping the leading term one gets finally for the divergent part

$$I_{\bar{n}}^{\pm i\eta} = -\frac{g^2}{4\pi^2}C_F \frac{1}{\varepsilon} [1 + \ln \mp i\eta] \quad (18)$$

and with a Principal Value (PV) prescription,

$$\text{Im} I_{\bar{n}}^{(PV)}(p) = \frac{1}{2}(\text{Im} I_{\bar{n}}^{(+i0)}(p) + \text{Im} I_{\bar{n}}^{(-i0)}(p)) = 0. \quad (19)$$

Thus all these prescriptions give results which depend on η and the imaginary part of the integral varies with the prescription. If one regulates the IR collinear singularity within pure dimensional regularization both $\text{Im} I_{\bar{n}}^{(+i0)}(p) = 0$ and $\text{Im} I_{\bar{n}}^{(-i0)}(p) = 0$ so that it seems that gauge invariance is achieved (also without the introduction of T). However this is a particular feature of an IR regulator. In principle the insertion of T and the use of the equivalent of Eq. 16, $I_{\bar{n},Fey} = -I_{w,Ax}^{(\text{Pres})}/2 + I_{T,Ax}^{(\text{Pres})}$, and $C_\infty^{(\text{Pres})}$ as given in Tab. II, removes completely the gauge changing imaginary part of the integrals. The T so restores gauge invariance with all IR regulators.

Now we consider the pure collinear part of the matrix element, i.e., the one after the zero-bin contribution is subtracted. The fundamental observation is that for $\pm i0$ and PV prescriptions, the zero-bin subtraction cancels completely all the dependence on the IR parameter $\pm i\eta$ leaving us with the result of Feynman gauge (see the Appendix for the zero-bin contributions). In other words the zero-bin is responsible of canceling all prescription and IR-regulator dependence in the case of the $\pm i0$, PV prescriptions. Thus we see that at least at one loop one does not really need to introduce the T -Wilson line in those prescriptions (although it is certainly possible to define a matrix element and its zero-bin subtraction both and separately gauge invariant with the use of T). However as we established in the previous section this is not the case in the ML prescription which is the only one reliably consistent with the quantization of QCD in light-cone gauge.

III. APPLICATIONS AND CONCLUSIONS

Until now we have considered the contribution from the T -Wilson line in one collinear direction however in many applications of SCET there are more than one collinear direction.

To make the discussion more concrete let us consider the quark form factor both in full QCD and in SCET. In full QCD it is well-known that the electromagnetic current $j^\mu = \bar{\psi}\gamma^\mu\psi$ is a conserved quantity and is gauge invariant under arbitrary $SU(3)$ gauge transformation. In SCET it has been established by different authors [11] that the full QCD quark form factor $\langle q(p_2)|j^\mu|q(p_1)\rangle$ factorizes into an incoming jet, outgoing jet and a soft function built out of two soft Wilson lines. However all those treatments were actually performed in Feynman gauge and not in light-cone gauge. The calculation in SCET in light-cone gauge can now be performed with the T -Wilson line that enters into both jets and one needs to introduce two light-cone gauge fixing conditions for each collinear jet (or collinear Lagrangian) and two different T Wilson lines again for each collinear direction, say T_n and $T_{\bar{n}}$ (and/or their Hermitian conjugates.) With these two transverse Wilson lines each of the two jets becomes gauge invariant in both regular and singular gauges. Thus using T 's the factorization of the full QCD quark form factor established in covariant gauges carries through straightforwardly also in light-cone gauge.

As we mentioned before the practical importance of the T -Wilson lines in SCET is that it allows us to write a gauge invariant definitions of the non-perturbative matrix elements appearing in the factorization theorems for certain cross-sections where fields are separated in the transverse direction.

As an example we take the TMDPDF in QCD and in SCET. In QCD the gauge invariant definition was first given in Ref. [12] and studied in light-cone gauge in Ref. [6]. In all these works it is pointed out how in QCD new kind of divergences appear when the fields entering the matrix elements are separated in the transverse direction. The TMDPDF occur for instance in SIDIS and here the complete factorization is achieved only in the presence of transverse links. In particular in Ref. [6] it is shown how to use light-cone gauge for practical calculations in SIDIS. Within SCET the TMDPDF for a quark q in a hadron P with momentum p can be defined with the use of $\underline{\chi}$ -field

$$\begin{aligned}\underline{\chi}_{\bar{n}}(y) &\equiv T_{\bar{n}}^\dagger(y^+, \mathbf{y}_\perp) W_{\bar{n}}^\dagger(y) \xi_{\bar{n}}(y), \\ \phi_{q/P} &= \langle P_{\bar{n}} | \underline{\chi}_{\bar{n}}(y) \delta\left(x - \frac{n\mathcal{P}}{np}\right) \delta^{(2)}(p_\perp - \mathcal{P}_\perp) \frac{\not{n}}{\sqrt{2}} \underline{\chi}_{\bar{n}}(0) | P_{\bar{n}} \rangle, \end{aligned} \quad (20)$$

where x is the momentum fraction of the quark in the light-cone direction, and \mathcal{P} is the usual label operators in SCET. The above definition of the TMDPDF is now gauge invariant in both classes of gauges. The complete analysis of TMDPDF in SCET and its application to SIDIS and other semi-inclusive processes will be developed in a forthcoming work [7].

The above analysis of the TMDPDF can be straightforwardly extended to consider the non-perturbative matrix elements introduced in [13–16] where one needs to invoke the T -Wilson line in the operator definition to obtain gauge invariant matrix elements as they should be. A final comment concerns the use of prescriptions in QCD. In ref. [9] it was argued that the only prescription that allows the quantization of QCD is the ML prescription. However in SCET we are considering just the Fock space of collinear particles and not the whole Fock space of QCD. The fact that we are considering a restriction of the whole Fock space implies that we have to re-discuss the quantization of the theory in this particular

subspace [17] with the help of SCET. Moreover in Ref. [9] the role of the T -Wilson line was not discussed. In Ref. [16] the authors use SCET in light-cone gauge and find that their calculation cannot give the correct result in the ML prescription (so contradicting the results of Ref. [9]). We think that the inclusion of the T -Wilson line would reconcile the calculation in Ref. [16] with the ML prescription [17].

To conclude we have shown that the introduction of the T -Wilson line in SCET is mandatory to achieve gauge invariant matrix elements both in covariant as well as in singular gauges. We explicitly performed one loop calculation for the SCET fundamental quantity $\langle 0 | W_{\bar{n}}^\dagger \xi_{\bar{n}} | q_{\bar{n}} \rangle$ in both Feynman and light-cone gauges and showed that in the ML prescription the T -Wilson line must be introduced to obtain gauge invariance. In other prescriptions commonly used in light-cone gauge, the soft or zero-bin subtractions remove the prescription dependence thus the purely collinear SCET matrix elements are gauge invariant in those prescriptions. However as we mentioned earlier such prescriptions may not lead to a proper quantization of SCET as it is the case in QCD.

The introduction of the T -Wilson line allows SCET to properly define in a gauge invariant way non-perturbative matrix elements such as the TMDPDF and generalizations thereof.

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Appendix

Considering the axial part of the WFR diagram the integral we need to calculate in $\pm i0$ prescriptions is

$$I_{\bar{n}}^{\pm i0} = -2ig^2 C_F \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i0)(k^+ \pm i0)} \frac{p^+ + k^+}{(p+k)^2 + i0} , \quad (21)$$

We regularize the IR divergence with ($\eta > 0$)

$$\begin{aligned} I_{\bar{n}}^{\pm i0} &= -2ig^2 C_F \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i0)(k^+ \pm ip^+ \eta)} \frac{p^+ + k^+}{(p+k)^2 + i0} \\ &= -2 \frac{\alpha_s}{4\pi} C_F \left(\frac{-p^2}{\mu^2} \right)^{-\epsilon} \Gamma(\epsilon) \int_0^1 dz z^{-\epsilon} (1-z)^{1-\epsilon} \frac{1}{z \mp i\eta} \\ &= -2 \frac{\alpha_s}{4\pi} C_F \left\{ \frac{1}{\epsilon} (-1 - \ln(\mp i\eta)) + \ln \left(\frac{-p^2}{\mu^2} \right) (1 + \ln(\mp i\eta)) + \frac{1}{2} \ln^2(\mp i\eta) - 2 + \frac{\pi^2}{3} \right\} \end{aligned} \quad (22)$$

This result is in agreement with Ref. [6] when η is small. The zero-bin part of this integral, that must be subtracted is

$$\begin{aligned}
I_{\bar{n},0}^{\pm i0} &= -2ig^2 C_F \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i0)(k^+ \pm ip^+ \eta)} \frac{p^+}{p^2 + k^- p^+ + i0} \\
&= -2 \frac{\alpha_s}{4\pi} C_F \left\{ \frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} \left(\ln(\mp i\eta) + \ln\left(\frac{-p^2}{\mu^2}\right) \right) + \frac{1}{2} \left(\ln(\mp i\eta) + \ln\left(\frac{-p^2}{\mu^2}\right) \right)^2 + \frac{\pi^2}{4} \right\}
\end{aligned} \tag{23}$$

Subtracting $I_{\bar{n},\pm i0} - I_{\bar{n},0}^{\pm i0}$ one recovers the result of A. Manohar in Ref. [11]. In the ML prescription, with the off-shellness that we have considered we have

$$\mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i0)(k^+ + ip^+ \eta \text{Sgn}(k^-))} \frac{p^+}{(p+k)^2 + i0} = 0. \tag{24}$$

For a general off-shellness the result can be found in ref. [10]. Also the corresponding zero-bin subtracted integral, with the off-shellness that we have chosen

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i0)(k^+ + ip^+ \eta \text{Sgn}(k^-))} \frac{p^+}{p^2 + k^- p^+ + i0} = 0. \tag{25}$$

When integrating in the complex k^+ -plane all poles in the integrand of Eq. (24) and Eq. (25) lie on the same side so the integrals give null results.

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